

Intrinsic and oscillated astrophysical neutrino flavor ratios revisited

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Abstract

The pp interactions taking place in the cosmos around us are a source of the astrophysical neutrinos of all the three flavors. In these interactions, the electron and the muon neutrinos mainly come from the production and the decay of the π^\pm mesons, whereas the tau neutrinos mainly come from the production and the decay of the D_S^\pm mesons. We estimate the three intrinsic neutrino flavor ratios for $1 \text{ GeV} \leq E \leq 10^{12} \text{ GeV}$ in the pp interactions and found them to be $1 : 2 : 3 \times 10^{-5}$. We study the effects of neutrino oscillations on these intrinsic ratios. We point out that the three ratios become $1 : 1 : 1$ if $L(\text{pc})/E(\text{GeV}) \geq 10^{-10}$ in the presence of neutrino oscillations, where L is the distance to the astrophysical neutrino source in units of parsecs.

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I. INTRODUCTION

Neutrino astronomy holds a great promise to explore the interiors of the dense astrophysical systems, which is not possible by any other existing means such as through the study of the cosmic-rays and/or the gamma-rays [1, 2, 3]. It is mainly because once the neutrinos are produced in the distant cosmos, they are essentially unobstructed by the intervening background matter mainly owing to their *weak* interaction cross section. On the other hand, the cosmic-rays (being the charged particles) and the gamma-rays are either deflected or even considerably absorbed by the same intervening background matter, depending upon the energy [4].

The presence of the proton component in the observed cosmic-ray flux for the entire energy range ($1 \text{ GeV} \leq E_p \leq 10^{12} \text{ GeV}$), may already be signaling the anticipated existence of the neutrino astronomy. The search of neutrinos from the cosmos may help to find a unified explanation for the common origin of the ultra-high energy cosmic-rays and the high energy gamma-rays. The absolute levels of the astrophysical neutrino productions are determined/affected by the intervening background matter [5, 6], the relative levels nevertheless do not. These thus remain important observables for the forthcoming detailed astrophysical neutrino searches. It is thus important to investigate the relative neutrino production levels in an astrophysical neutrino source; as well as the changes that may occur in the relative composition of the neutrino flavors during their propagation to us.

Given the recent empirical evidences of the **neutrino oscillations** [7], it is timely to perform a reference study for the effects of the neutrino oscillations on the mixed intrinsic astrophysical neutrino flavor ratios during their propagation. Further motivation is provided by the recent developments both in providing examples to use the astrophysical neutrinos not only to study the cosmos around us [8, 9, 10, 11], but also to explore the properties of the neutrinos itself (including those suggested beyond the Standard Model of Particle Physics) [12].

We consider the $p p$ interactions as a source of the intrinsic astrophysical neutrino production. The first p represent the cosmic-ray flux produced inside the source, whereas the second p represent the medium contribution in the source. Commonly cited examples of the astrophysical systems where the $p p$ interactions play a role include the nearby astrophysical sources such as the earth atmosphere and the galactic plane/center region as well as other

sources inside our galaxy. The more distant suggested astrophysical neutrino sources include the Active Galactic Nuclei (AGNs) [13], and the sites of the Gamma-Ray Bursts (GRBs) [14].

We estimate the three intrinsic neutrino flavor ratios as a function of the neutrino energy and study the neutrino oscillation effects on these for neutrino energy ranging between 1 GeV and 10^{12} GeV. The **energy dependence** of the three intrinsic ratios has not been studied previously. We mainly investigate the particle physics aspects of the three ratios. A purpose of the present study is to provide a firm basis for the relevance of the neutrino oscillation effects for the forthcoming searches of the three astrophysical neutrino flavor ratios. As in order to determine the astrophysical source characteristics as precisely and as completely as possible via neutrinos, one needs to know the relevance of the distance to the source L in the **presence** of the neutrino oscillations for the given neutrino energy E , since the neutrino oscillations redistribute the intrinsic neutrino flavor ratios depending upon the value of the ratio L/E .

This paper is organized as follows. In Section II, we estimate the three intrinsic astrophysical neutrino flavor ratios in some detail mainly within the framework of the Quark-Gluon String Model (QGSM). In Section III, we study the effects of the neutrino oscillations on these. We identify the range of the L/E values where the commonly considered assumption of the averaging of neutrino oscillation probabilities may hold. In Section IV, we briefly summarize the present status of the relevant detection strategies as well as the detector developments to search for the astrophysical neutrino flavor ratios. In Section V, we present our conclusions.

II. THE INTRINSIC ASTROPHYSICAL NEUTRINO FLAVOR RATIOS

Let us define the three intrinsic neutrinos flavor ratios as the $R_{e/e}^0$, the $R_{\mu/e}^0$, and the $R_{\tau/e}^0$. Here $R_{\tau/e}^0 \equiv F_{\nu_\tau}^0(E)/F_{\nu_e}^0(E)$ with $F_{\nu_\tau}^0(E) = dN_{\nu_\tau}^0/dE$, for instance. Clearly, $R_{e/e}^0 = 1$. This ratio provides normalization for the other two ratios. The previous estimates for the $R_{\tau/e}^0$ ratio are between $\geq 10^{-4}$ [15] and $\geq 10^{-5}$ [16]. However, the energy dependence of the three intrinsic neutrino flavor ratios was not studied.

We use the following formula for computing the astrophysical neutrino flux spectrum

$$F_\nu^0(E) = \frac{dN_\nu^0}{dE} = dn_p \int_E^\infty dE_p \phi_p(E_p) \frac{d\sigma_{pp \rightarrow \nu+Y}}{dE}, \quad (1)$$

where E is the neutrino energy. The cosmic-ray flux spectrum, $\phi_p(E_p)$, is given by [17]

$$\phi_p(E_p) = A(E_p/\text{GeV})^\delta \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1}, \quad (2)$$

where $A = 0.8$ and $\delta = -2.75$. We use the above cosmic-ray flux spectrum for $1 \text{ GeV} \leq E_p \leq 10^{12} \text{ GeV}$. We are aware that the above cosmic-ray flux spectrum differs by few percent for $E_p \leq 10 \text{ GeV}$ as compared to the more recent compilation [18]. However, this is of not much concern for our present study since we are primarily interested in studying the **astrophysical neutrino flavor ratios** here. Moreover, later, we also study the effects of varying the exponent δ . In the above simplified picture, it is assumed that all the hadrons and the relevant leptons decay before interacting with the medium of the astrophysical neutrino source.

The representative proton number density inside the source is taken to be $n_p = 1 \text{ cm}^{-3}$, and the representative distance d inside the source is taken to be $\sim 10 \text{ kpc}$, where $1 \text{ pc} \simeq 3 \times 10^{18} \text{ cm}$. These values are to merely represent the reference absolute levels for the three neutrino fluxes. As stated earlier, our main concern in this paper is to study their ratios and their energy dependence defined earlier which are obviously independent of the product dn_p . It is clear that the task of computing the dN_ν^0/dE in Eq. (1) essentially relies on the evaluation of the differential cross section $d\sigma/dE$ in the pp interactions.

In this work, we shall consider only the most dominant production channels for each neutrino flavor generation as the representative examples. Namely, the π^\pm meson for the electron and the muon neutrinos and the D_s^\pm meson for the tau neutrinos. We employ the Quark-Gluon String Model (QGSM) to calculate the production distributions of the above mesons. The QGSM approach is non-perturbative and is based on the string fragmentation. It contains a number of parameters determined by the experiments [19, 20, 21, 22, 23].

The production cross section of the hadron h in the QGSM is given by

$$\frac{d\sigma^h(s, x)}{dx} \approx \frac{1}{\sqrt{x^2 + x_\perp^2}} \left[\sum_{n=1}^{\infty} \sigma_n^{pp}(s) \phi_n^h(s, x) + \sigma_{DD}(s) \phi_0^h(s, x) \right], \quad (3)$$

where $x = 2p_\parallel^h/\sqrt{s}$ and $x_\perp = 2\sqrt{(m_h^2 + p_\perp^{h2})/s}$. The p_\parallel^h (p_\perp^h) is the parallel (perpendicular) momentum of the secondary hadron h in the center of mass frame. All the related formulas are provided in the Appendix.

We have used the VEGAS multi-dimensional Monte Carlo integration program [24, 25] to obtain the energy distribution of the neutrino flux by selecting the events falling inside the considered energy segment through a series of boosts to the laboratory frame. We have simplified the secondary decays of the charged leptons, the μ and the τ , as effectively the two body ones, that is, a neutrino plus a particle with varying invariant mass. For instance, we assume that the τ lepton decays into a ν_τ and a particle Y with the mass m_Y satisfying $0.1 \text{ GeV} < m_Y < m_\tau - 0.1 \text{ GeV}$. In our setting $s \sim 2m_p E_p$. We take the $D_S^+ \rightarrow \tau^+ \nu_\tau$ branching ratio as ~ 0.064 [26].

We have calculated the following processes in the QGSM:

$$\begin{aligned} pp \rightarrow \pi^\pm (\rightarrow \mu \nu_\mu) + X \\ \rightarrow e \nu_e \nu_\mu, \\ pp \rightarrow D_s^\pm (\rightarrow \tau \nu_\tau) + X \\ \rightarrow \nu_\tau + Y. \end{aligned} \tag{4}$$

The tau neutrino production via the D_s^\pm meson can also be calculated in the perturbative Quantum Chromo Dynamics (pQCD) through $pp \rightarrow c\bar{c} \rightarrow D_s^\pm + X \rightarrow \nu_\tau + Y$. However, the electron and the muon neutrino production through the π^\pm meson can not be handled in the pQCD. The reason is as follows. There are many quark-level sub-processes (the t -, the u - as well as the s -channel) for the π^\pm production in the pp interactions, differently from the D_s^\pm production which has only the $c\bar{c}$ pair production channel. Without some cuts such as on the p_T or some cuts on the factorization scale etc., the light quark productions may blow up as they approach the non-perturbative region.

In the pQCD calculations, we use the leading-order results of the parton sub processes $q\bar{q}, gg \rightarrow Q\bar{Q}$ where $Q = c$ for $pp \rightarrow (c\bar{c} \rightarrow D_s^\pm \rightarrow) \nu_\tau Y$ [and $Q = t$ for $pp \rightarrow t\bar{t} \rightarrow \nu X$]. We use a K factor, $K = 2$, to account for the NLO corrections. For the parton distribution functions, we use the CTEQv6 [27, 28]. We use $m_c = 1.35 \text{ GeV}$, $m_t = 175 \text{ GeV}$, α_s with $\alpha_s(M_Z^2) = 0.118$ and $Q^2 = \hat{s}/4$ as a factorization scale. For the ν_τ production through the $c\bar{c}$, we use the Peterson fragmentation function with $\epsilon \approx 0.029$ for fragmentation of c or \bar{c} into the D_s^\pm meson [29, 30].

Fig. 1 shows the three intrinsic neutrino flavor fluxes as a function of the neutrino energy E . We plot $dN_\nu^0/d(\log_{10} E)$ in units of $\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}$ as a function of the E . Since the tau neutrino production via the D_S^\pm can be dealt with in both the pQCD and the QGSM, the

pQCD result for the tau neutrinos is also presented for comparison. We note that there is a relatively large discrepancy at higher energy ($E \geq 10^9$ GeV) between the two approaches for the ν_τ production.

To consider an example of the process that may produce the three neutrino flavors without hadronizing in the pp interactions, we have studied the $pp \rightarrow t\bar{t}$ channel. Here, the neutrino energy distributions are not effected by the hadronization process. The neutrino production in the $pp \rightarrow t\bar{t} \rightarrow \nu X$ is reliably calculable in the pQCD. Here, we consider the direct decays of t into each lepton (with $\sim 10\%$ branching ratio) and include the secondary decays of massive leptons (μ and τ). Note that all the lepton masses are very small as compared to the top quark mass, as a result all the three neutrino distributions are of the same orders of magnitude. These results are also shown in the Fig. 1. It is clear from the figure that the three neutrino **fluxes** are comparable for this process. Compared to the π^\pm and the D_S^\pm results, the $t\bar{t}$ contributions are negligible, less than a factor of at least $\sim 10^{-3}$ over the whole range of the considered energies.

It can also be seen from the figure that the production of the π^\pm , the D_s^\pm , and their branching ratios of relevant leptonic decays are more important than the effects of the lepton masses. Our results are in good agreement for the ν_e and the ν_μ production with those given in the Ref. [31] using the PYTHIA, whereas for the ν_τ production, our results are in good agreement with those given in the Ref. [32].

Fig. 2 shows the three intrinsic neutrino flavor ratios defined earlier, as a function of the neutrino energy E . We note that an **energy independent** relative flux hierarchy among the three intrinsic neutrino flavor ratios persisted even at the highest considered energy, namely $R_{e/e}^0 : R_{\mu/e}^0 : R_{\tau/e}^0 \propto 1 : 2 : 3 \times 10^{-5}$ for $1 \text{ GeV} \leq E \leq 10^{12} \text{ GeV}$. Our results are however subject to the uncertainties of extrapolating the parameters of the hadron/quark production models especially for $E \geq 10^6$ GeV. This corresponds to center-of-mass energy of $\sqrt{s} \geq 10^3$ GeV. To our knowledge, the QGSM parameters are fitted up to $\sqrt{s} \sim 540$ GeV using the SPS collider data for the light mesons, whereas for the charmed mesons the comparison is available using the $\sqrt{s} \sim 630$ GeV data.

In Fig. 3, we show the energy dependence of the two intrinsic ratios of the astrophysical neutrino fluxes, the $R_{\mu/e}^0$ and the $R_{\tau/e}^0$, by varying the exponent δ of the cosmic-ray flux spectrum, $\phi_p(E_p)$ [see Eq. (2)], in the range $-1.75 \leq \delta \leq -3.75$. It is so because the cosmic-ray flux spectrum in an astrophysical source is expected to be harder than the observed one

locally [17]. We note that the ratio $R_{\tau/e}^0$ changes from $\sim 3 \times 10^{-5}$ to $\sim 8 \times 10^{-5}$, when the δ changes from -2.75 to -1.75 for $1 \text{ GeV} \leq E \leq 10^{12} \text{ GeV}$. However, except the above (slight) change, in general, we note from the figure that the two intrinsic neutrino flavor ratios are essentially stable w.r.t. the δ variation.

For an example of possible energy dependence in the the three astrophysical neutrino flavor ratios coming from astrophysical reasonings, see [33].

III. THE OSCILLATED ASTROPHYSICAL NEUTRINO FLAVOR RATIOS

In this Section, we shall perform a three neutrino oscillation analysis of the three intrinsic neutrino flavor ratios estimated in the previous Section. In the context of the three neutrino flavors, there are 6 independent neutrino mixing parameters. The matter effects are found to be negligible for the entire range of the neutrino mixing parameters and the E values under discussion [34, 35].

In this analysis, we do not make the assumption of averaging over the neutrino oscillation probabilities. The neutrino oscillation effects for the astrophysical neutrinos, using the averaged oscillation probability expressions, were studied in some detail in Ref. [36]. Here, instead, we shall determine the L/E range that may be relevant for the averaging.

We start with the connection U between the flavor $|\nu_\alpha\rangle$ and the mass $|\nu_i\rangle$ eigenstates of the neutrinos, namely

$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i} |\nu_i\rangle, \quad (5)$$

where $\alpha = e, \mu, \tau$. In the context of the three neutrinos, U is called the Maki Nakagawa Sakita (MNS) mixing matrix [37]. Under the assumption that the CP violating phase $\delta_{CP} = 0$, the 3×3 MNS mixing matrix U in the standard parameterization connecting the neutrino mass and the flavor eigenstates reads [26]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \quad (6)$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. The presently available empirical constraints for the various neutrino mixing parameters, in the context of the three neutrino mixing, give the

following best fit values: $\theta_{12} = 33.2^\circ$, $\theta_{23} = 45.0^\circ$, $\theta_{13} = 0.0^\circ$, $\delta m_{12}^2 = 7.9 \times 10^{-5}$ eV² and $\delta m_{23}^2 = 2.1 \times 10^{-3}$ eV² [7].

Using Eq. (6), the neutrino oscillation probability formula is [38]

$$P(\nu_\alpha \rightarrow \nu_\beta; L, E) \equiv P_{\alpha\beta}(L, E) = \sum_{i=1}^3 U_{\alpha i}^2 U_{\beta i}^2 + \sum_{i \neq j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \cos \left(\frac{2L}{L_{ij}} \right), \quad (7)$$

where $\beta = e, \mu, \tau$ and $L_{ij} \simeq 4E/\delta m_{ij}^2$ is the neutrino oscillation length. The L in Eq. (7) is the neutrino flight length.

The neutrino flux ratios $R_{\alpha/\beta}(L, E)$, arriving at the detector, in the presence of neutrino oscillations are estimated using the relation

$$F_{\nu_\alpha}(L, E) = \sum_{\beta} P_{\alpha\beta}(L, E) F_{\nu_\beta}^0, \quad (8)$$

and the definition $R_{\alpha/\beta} = F_{\nu_\alpha}(E)/F_{\nu_\beta}(E)$, so that

$$R_{\alpha/\beta}(L, E) = \frac{P_{\alpha e}(L, E) + P_{\alpha \mu}(L, E) R_{\mu/e}^0 + P_{\alpha \tau}(L, E) R_{\tau/e}^0}{P_{\beta e}(L, E) + P_{\beta \mu}(L, E) R_{\mu/e}^0 + P_{\beta \tau}(L, E) R_{\tau/e}^0}, \quad (9)$$

where the $R_{\alpha/\beta}^0$ are taken according to the discussion in the previous Section. The $P_{\alpha\beta}(L, E)$ is obtainable using Eq. (7). The unitarity conditions such as $1 - P_{ee}(L, E) = P_{e\mu}(L, E) + P_{e\tau}(L, E)$ are implemented at each L and E at which these are evaluated.

Fig. 4 shows the three oscillated neutrino flavor ratios as a function of the ratio L/E . Here, we have used the fact that the three intrinsic ratios are essentially independent of the L as well as the E . Note that the ratio $R_{\tau/\mu}$ behaves like the ratio $R_{\tau/e}$. The ratio $R_{e/e}$ is not plotted as it is unaffected by the neutrinos oscillations. From the figure, it is clear that the averaging may be a good approximation if $L(\text{pc})/E(\text{GeV}) \geq 10^{-10}$, namely

$$1 : 2 : 3 \times 10^{-5} \longrightarrow \nu \text{ osc and if } L(\text{pc})/E(\text{GeV}) \geq 10^{-10} \longrightarrow 1 : 1 : 1. \quad (10)$$

Fig. 5, which is our main result, shows the region in the L versus E plane where the averaging of the neutrino oscillations probabilities may be assumed, under the above criterion. For instance, if $E \sim 10^4$ GeV, and if the distance to the source is $\geq 10^{12}$ cm then the incoming astrophysical neutrino flux should be essentially an equal admixture of the three neutrino flavors.

IV. PROSPECTS FOR ASTROPHYSICAL NEUTRINO FLAVOR IDENTIFICATION

In this Section, we shall briefly summarize the presently envisaged detection strategies and the detector configurations for the possible astrophysical neutrino flavor identification for $1 \text{ GeV} \leq E \leq 10^{12} \text{ GeV}$ mainly in the context of the Cherenkov radiation detection. For a discussion of other alternative detection strategies such as using the radio and the acoustic signals, see Ref. [39, 40].

From the reference estimates presented in Section II, it follows that the flux of the intrinsic astrophysical tau neutrino flavor is considerably suppressed for $1 \text{ GeV} \leq E \leq 10^{12} \text{ GeV}$ relative to that of the electron and muon neutrino flavor. According to the discussion in Section III, the neutrino oscillations populate the astrophysical tau neutrino flux comparable to the astrophysical electron and the muon neutrino fluxes, provided the ratio L/E is in a certain range. A signature of the neutrino oscillations in the mixed astrophysical neutrino flux thus shall be the **identification of the astrophysical tau neutrino flavor** among the other two. A commonly used essential ingredient in this context is to make a comparative use of the characteristic energy dependent astrophysical tau neutrino induced tau lepton decay and/or interaction length scale relative to the relevant electron and the muon length scales [41, 42, 43].

A. Current detection strategies

The astrophysical neutrino detection can be achieved in the neutrino nucleon/electron interactions [44, 45]. These interactions may occur near or inside the detector. The charged leptons, the (air) showers, and the associated radiations such as the radio and/or acoustic signals are the measurable quantities. The detectors are optimized for their performance in discriminating the three neutrino flavors in certain energy intervals [46]. In addition to reconstructing the astrophysical neutrino flavor ratios from an individual detector, comparing the data from the various detectors shall also lead to the possibility of identification of the three neutrino flavor ratios. For a recent discussion of the event rates in some representative astrophysical neutrino flux models, see Ref. [47]. The astrophysical neutrinos arrive at an earth based detector in the three general directions.

The downward going astrophysical neutrinos do not traverse any significant cord length inside the earth to reach the detector. In fact, for large earth surface shower detectors, the neutrino nucleon interactions take place in the earth atmosphere and the resulting shower (or part of it) is observed by the detectors. Several studies were performed to identify the detector specifications and/or the energy ranges in which a specific detector configuration is/can be optimized to disentangle the astrophysical tau neutrino flavor from the astrophysical electron and/or muon neutrino flavor through the *double shower* or *single shower* event topologies [15, 48, 49, 50, 51].

The upward going neutrinos traverse a large earth cord before they reach the detector. At energies $E_0 \geq 5 \times 10^4$ GeV, the charged current neutrino nucleon interaction length is smaller than the earth diameter. As a result significant **neutrino flavor dependent absorption** takes place for $E \geq E_0$. For a discussion of the upward going tau neutrino flavor behavior versus the upward going muon neutrino flavor, see Ref. [52, 53].

It might also be possible to search for the air showers induced by the incoming (quasi-horizontal) astrophysical neutrinos, in case the neutrinos happen to have just one interaction inside the earth. This strategy is referred to as the earth skimming [54]. In some configurations, the astrophysical neutrinos may interact just below (within $5^\circ \sim 10^\circ$ of) the detector horizon [55, 56, 57, 58, 59, 60, 61]. The air showers produced by the earth skimming astrophysical neutrinos can also be searched by the forthcoming large scale balloon/space based detectors as well [62].

B. Present and the forthcoming detectors

Presently operating detectors searching for the astrophysical neutrinos include the Antarctic Muon and Neutrino Detector Array (AMANDA), its proposed extension, the Ice cube [63, 64, 65], and the lake Baikal detector [66]. These detectors use the ice and the water as detection medium respectively, and are sensitive to all the three neutrino flavors essentially for $10^3 \text{ GeV} \leq E \leq 10^6 \text{ GeV}$ and mainly search for the upward going (and horizontal) neutrinos. Other under construction large scale detectors include the Astronomy with a Neutrino Telescope and Abyss environmental REsearch (ANTARES) project [67]. For $E \leq 10^3$ GeV, the Superkamiokande and the upcoming one Mega ton class of detectors shall be sensitive to the three neutrino flavors [68].

Several attempts are underway to implement the earth skimming strategy using the mountains as target for the neutrino nucleon/electron interactions, such as the concept study carried out by the Neutrino Telescope (NuTel) collaboration [69]. This class of detectors are/shall be essentially sensitive for $10^6 \text{ GeV} \leq E < 10^8 \text{ GeV}$, mainly for the tau neutrino flavor. Earth/air/sea skimming tau neutrino air shower search for more wider energy range, namely for $10^6 \text{ GeV} < E \leq 10^{10} \text{ GeV}$, is also recently suggested [70, 71].

The under construction large surface array detectors such as the Pierre Auger (PA) observatory [72, 73], and the Telescope Array (TA) experiment shall also be sensitive to all the three neutrino flavors for $10^7 \text{ GeV} < E < 10^{11} \text{ GeV}$ [74]. Orbiting Wide-field Light collector space based mission (OWL) shall be sensitive to the $\nu_e(\bar{\nu}_e)$ flavor for $10^{10} \text{ GeV} < E \leq 10^{12} \text{ GeV}$ as it shall search for the atmospheric fluorescent trail using earth atmosphere as the detection medium in the neutrino nucleon interactions [75]. The Extreme Universe Space Observatory (EUSO) shall be sensitive to all the three neutrino flavors by detecting the fluorescent and the Cherenkov light produced in the air showers generated by the neutrino nucleon interaction occurring in the earth atmosphere for $10^{10} \text{ GeV} < E \leq 10^{12} \text{ GeV}$ [76].

The detectors based on the alternative techniques are also taking data. These include the Radio Cherenkov Experiment (RICE) that is sensitive to the $\nu_e(\bar{\nu}_e)$ flavor based on the anticipated radio-wavelength Cherenkov radiation detection that shall be produced by the neutrino nucleon interactions in the polar ice for $10^7 \text{ GeV} < E \leq 10^{12} \text{ GeV}$ [77, 78]. The high altitude balloon based Antarctic Impulsive transient Antenna (ANITA) experiment shall search for the ice skimming $\nu_e(\bar{\nu}_e)$ flavor induced coherent radio signals for $E \geq 10^9 \text{ GeV}$ [79]. For $E \geq 10^{11} \text{ GeV}$, upper limits are also provided by the the Goldstone Lunar Ultra-high energy neutrino Experiment (GLUE) mainly for the $\nu_e(\bar{\nu}_e)$ flavor, based on the similar detection technique [80]. For a summary of upper limits based on the alternative detection methods for $E > 10^9 \text{ GeV}$, see [81].

V. CONCLUSIONS

We have performed a reference estimate of the three intrinsic astrophysical neutrino flavor ratios for the neutrino energy ranging between 1 GeV and 10^{12} GeV in the pp interactions mainly within the framework of the Quark-Gluon String Model (QGSM). We have taken into

account the π^\pm meson production for the generation of the ν_e and the ν_μ neutrino flavors, and the D_S^\pm meson production for the generation of the ν_τ neutrino flavor. We have also studied the $t\bar{t}$ production using the perturbative Quantum Chromo Dynamics (pQCD) as an example of the process for the generation of the three neutrino flavors **without hadronization** in the pp interactions. The neutrino generation from the latter channel is found to be suppressed for the entire considered energy range.

We have only taken into account the proton component in the observed cosmic-ray flux spectrum. We have also studied the variation of the cosmic-ray flux spectrum exponent δ on the three ratios and found that the three intrinsic ratios are essentially independent of the exponent for $-1.75 \leq \delta \leq -3.75$.

The three astrophysical neutrino flavor ratios are essentially **independent of energy** and are $1 : 2 : 3 \times 10^{-5}$ for $1 \text{ GeV} \leq E \leq 10^{12} \text{ GeV}$. Namely, the relative intrinsic neutrino flux hierarchy stays the same for the entire considered energy range. Therefore, the intrinsic astrophysical tau neutrino flavor is relatively suppressed for the entire considered energy range. Our considered energy range covers the entire E range of the observed cosmic-ray flux.

We have studied the **effects of the neutrino oscillations** in the three neutrino flavor framework on the three intrinsic ratios, using the recent best fit values of the neutrino mixing parameters. The neutrino oscillation effects depend upon the distance to the astrophysical source L for the given neutrino energy E . For $L(\text{pc})/E(\text{GeV}) \geq 10^{-10}$, the averaging of the neutrino oscillation probabilities may be assumed, where the L is in the units of parsecs. Our present estimate is intended to provide a firm basis for the relevance of the **neutrino oscillations effects** for the forthcoming search of the astrophysical neutrino flavor ratios by the various detectors.

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APPENDIX A: FORMULAS FOR THE QGSM

In Eq. (3), the functions σ_n^{pp} and $\phi_n^h(s, x)$ are defined as follows:

$$\begin{aligned}\phi_n^h(s, x) &= a^h [F_{qq}^h(x_+, n) F_q^h(x_-, n) + F_q^h(x_+, n) F_{qq}^h(x_-, n) \\ &\quad + 2(n-1) F_{sea}^h(x_+, n) F_{sea}^h(x_-, n)] \quad \text{for } n \geq 1, \\ \phi_0^h(s, x) &= \frac{3}{2} a^h (\sqrt{x_+} + \sqrt{x_-}) F_q^h(x_+, 1) F_q^h(x_-, 1),\end{aligned}$$

where $x_\pm = (\sqrt{x^2 + x_\perp^2} \pm x)/2$ and

$$F_q^h(x, n) = \frac{2}{3} \int_x^1 dx_1 f_p^u(x_1, n) G_u^h\left(\frac{x}{x_1}\right) + \frac{1}{3} \int_x^1 dx_1 f_p^d(x_1, n) G_d^h\left(\frac{x}{x_1}\right), \quad (\text{A1})$$

$$F_{qq}^h(x, n) = \frac{2}{3} \int_x^1 dx_1 f_p^{ud}(x_1, n) G_{ud}^h\left(\frac{x}{x_1}\right) + \frac{1}{3} \int_x^1 dx_1 f_p^{uu}(x_1, n) G_{uu}^h\left(\frac{x}{x_1}\right), \quad (\text{A2})$$

$$\begin{aligned}F_{sea}^h(x, n) &= \frac{1}{4+2\delta_s} \left\{ \int_x^1 dx_1 f_p^{usea}(x_1, n) \left[G_u^h\left(\frac{x}{x_1}\right) + G_{\bar{u}}^h\left(\frac{x}{x_1}\right) \right] \right. \\ &\quad + \int_x^1 dx_1 f_p^{dsea}(x_1, n) \left[G_d^h\left(\frac{x}{x_1}\right) + G_{\bar{d}}^h\left(\frac{x}{x_1}\right) \right] \\ &\quad \left. + \delta_s \int_x^1 dx_1 f_p^{sssea}(x_1, n) \left[G_s^h\left(\frac{x}{x_1}\right) + G_{\bar{s}}^h\left(\frac{x}{x_1}\right) \right] \right\}. \quad (\text{A3})\end{aligned}$$

In the above equations, $f_p^i(x, n)$'s are the distribution functions describing the n -Pomeron distribution functions of quarks or diquarks ($i = u, d, uu, \dots$) with a fraction of energy x from the proton, and $G_i^h(z)$'s are the fragmentation functions of the quark or diquark chain into a hadron h which carries a fraction z of its energy.

The list of the $f_p^i(x, n)$ is as follows

$$\begin{aligned}f_p^u(x, n) &= \frac{\Gamma(1+n-2\alpha_N)}{\Gamma(1-\alpha_R) \Gamma(\alpha_R-2\alpha_N+n)} \times x^{-\alpha_R} (1-x)^{\alpha_R-2\alpha_N+(n-1)}, \\ &= f_p^{usea}, \\ f_p^d(x, n) &= \frac{\Gamma(2+n-2\alpha_N)}{\Gamma(1-\alpha_R) \Gamma(\alpha_R-2\alpha_N+n+1)} \times x^{-\alpha_R} (1-x)^{\alpha_R-2\alpha_N+n}, \\ &= f_p^{dsea}, \\ f_p^{uu}(x, n) &= \frac{\Gamma(2+n-2\alpha_N)}{\Gamma(-\alpha_R+n) \Gamma(\alpha_R-2\alpha_N+1)} \times x^{\alpha_R-2\alpha_N+1} (1-x)^{-\alpha_R+(n-1)}, \\ f_p^{ud}(x, n) &= \frac{\Gamma(1+n-2\alpha_N)}{\Gamma(-\alpha_R+n) \Gamma(\alpha_R-2\alpha_N+2)} \times x^{\alpha_R-2\alpha_N} (1-x)^{-\alpha_R+(n-1)}, \\ f_p^{sssea}(x, n) &= \frac{\Gamma(1+n+2\alpha_R-2\alpha_N-2\alpha_\phi)}{\Gamma(1-\alpha_\phi) \Gamma(2\alpha_R-2\alpha_N+n-\alpha_\phi)} \times x^{-\alpha_\phi} (1-x)^{2\alpha_R-2\alpha_N+(n-1)-\alpha_\phi},\end{aligned}$$

where Γ is the usual Gamma function.

The list of the $G_i^{D_s^\pm}(z)$ is given by

$$\begin{aligned} G_{u,\bar{u},d,\bar{d}}^{D_s^\pm}(z) &= (1-z)^{\lambda-\alpha_\psi+2-\alpha_R-\alpha_\phi}, \\ G_{uu,ud}^{D_s^\pm}(z) &= (1-z)^{\lambda-\alpha_\psi+\alpha_R-2\alpha_N-\alpha_\phi+2}, \\ G_s^{D_s^+}(z) &= (1-z)^{\lambda-\alpha_\psi+2(1-\alpha_\phi)}, \\ &= G_{\bar{s}}^{D_s^-}(z), \\ G_s^{D_s^-}(z) &= (1-z)^{\lambda-\alpha_\psi} \times (1+a_1 z^2), \\ &= G_{\bar{s}}^{D_s^+}(z). \end{aligned}$$

The list of the $G_i^{\pi^\pm}(z)$ is given by

$$\begin{aligned} G_u^{\pi^+}(z) &= (1-z)^{-\alpha_R+\lambda} \\ &= G_d^{\pi^+}(z) = G_d^{\pi^-}(z) = G_{\bar{u}}^{\pi^-}(z), \\ G_d^{\pi^+}(z) &= (1-z)^{-\alpha_R+\lambda+1} \\ &= G_{\bar{u}}^{\pi^+}(z) = G_u^{\pi^-}(z) = G_d^{\pi^-}(z), \\ G_{uu}^{\pi^+}(z) &= (1-z)^{\alpha_R-2\alpha_N+\lambda} \\ &= G_{ud}^{\pi^+}(z), \\ G_{uu}^{\pi^-}(z) &= (1-z)^{\alpha_R-2\alpha_N+\lambda+1} \\ &= G_{ud}^{\pi^-}(z). \end{aligned}$$

In the above, the input parameters are as follows:

$$\alpha_R = 0.5, \quad \alpha_N = -0.5, \quad \alpha_\phi = 0, \quad \alpha_\psi = -2.18, \quad \lambda = 0.5, \quad a_1 = 5$$

$$\delta_s = 0.25 \text{ (0), if strange sea contribution is turned on (off),}$$

$$a^{D_s} = 0.0007, \quad a^\pi = 0.44.$$

The function $\sigma_n^{pp}(s)$ and $\sigma_{DD}(s)$ are given by the following formulas:

$$\begin{aligned} \sigma_n^{pp}(\xi) &= \frac{\sigma_p}{n z} \left(1 - \exp(-z) \sum_{k=0}^{n-1} \frac{z^k}{k!} \right), \\ \sigma_{DD}(\xi) &= \frac{C-1}{C} \sigma_p [f(z/2) - f(z)], \end{aligned}$$

where

$$\xi = \ln \left(\frac{s}{1 \text{ (GeV)}^2} \right), \quad z = \frac{2C \gamma_p}{R^2 + \alpha_p' \xi} \exp(\xi \Delta), \quad \sigma_p = 8\pi \gamma_p \exp(\xi \Delta),$$

and

$$f(z) = \sum_{\nu=1}^{\infty} \frac{(-z)^{\nu-1}}{\nu\nu!} = \frac{1}{z} \int_0^z dx \frac{1-e^{-x}}{x}.$$

The best fit parameter values are as follows :

(i) for $\sqrt{s} \leq 10^3$ GeV

$$\gamma_p = 3.64 \text{ (GeV)}^{-2}, \quad R^2 = 3.56 \text{ (GeV)}^{-2}, \quad \alpha'_p = 0.25 \text{ (GeV)}^{-2},$$

$$C = 1.5, \quad \Delta = 0.07.$$

(ii) for $\sqrt{s} \geq 10^3$ GeV

$$\gamma_p = 1.77 \text{ (GeV)}^{-2}, \quad R^2 = 3.18 \text{ (GeV)}^{-2}, \quad \alpha'_p = 0.25 \text{ (GeV)}^{-2},$$

$$C = 1.5, \quad \Delta = 0.139.$$

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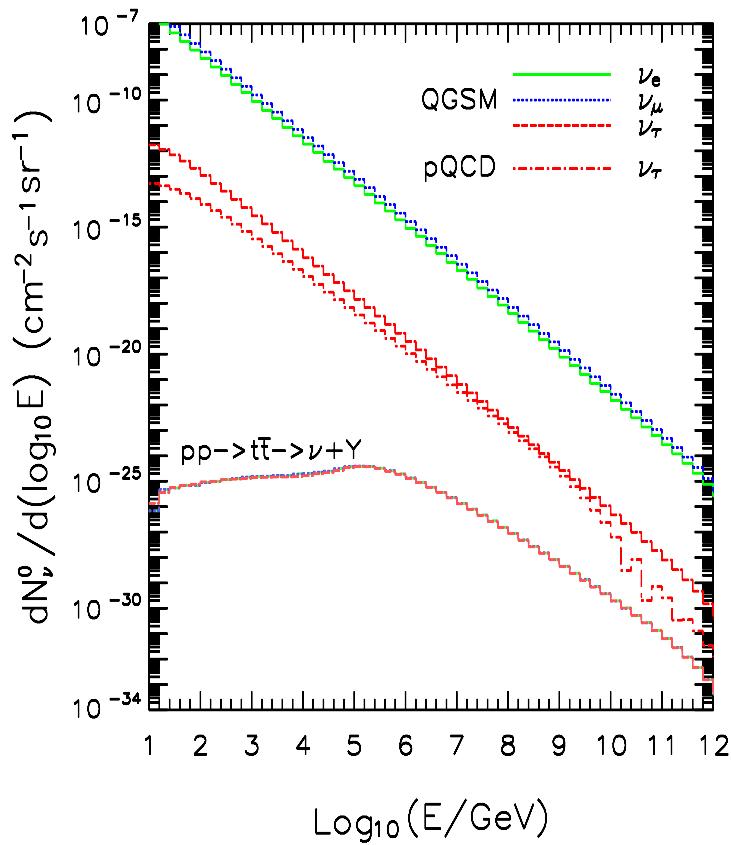


FIG. 1: The three neutrino fluxes in the QGSM in the pp interactions as a function of the neutrino energy E . For the tau neutrino, the pQCD result is also presented for comparison. The three neutrino fluxes in the $pp \rightarrow t\bar{t} \rightarrow \nu + X$, where t is the top quark, calculated in the pQCD, are also shown as a function of the neutrino energy E .

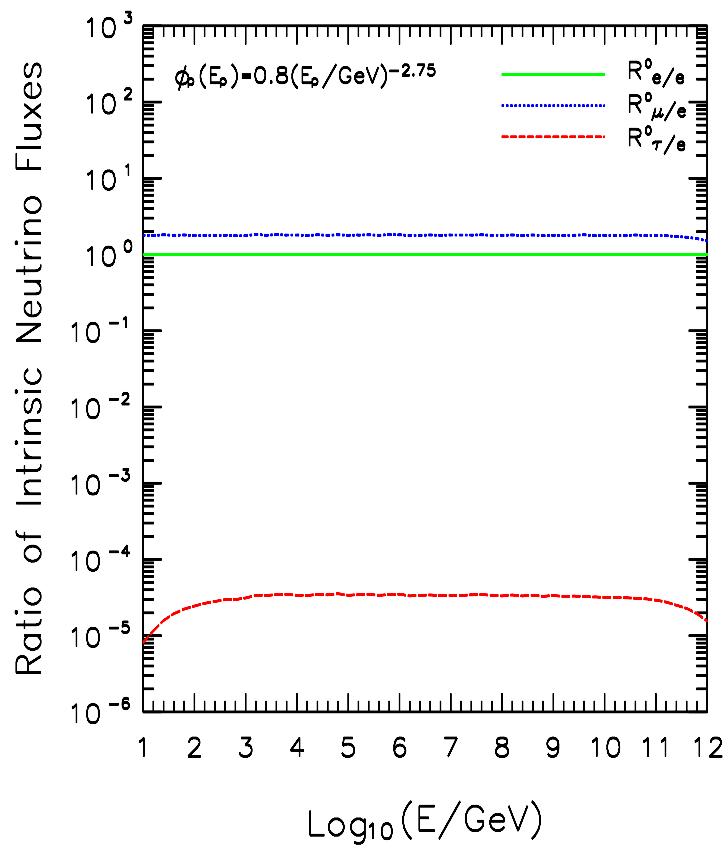


FIG. 2: The intrinsic astrophysical neutrino flavor ratios as a function of the neutrino energy E in the pp interactions. More details are given in the text.

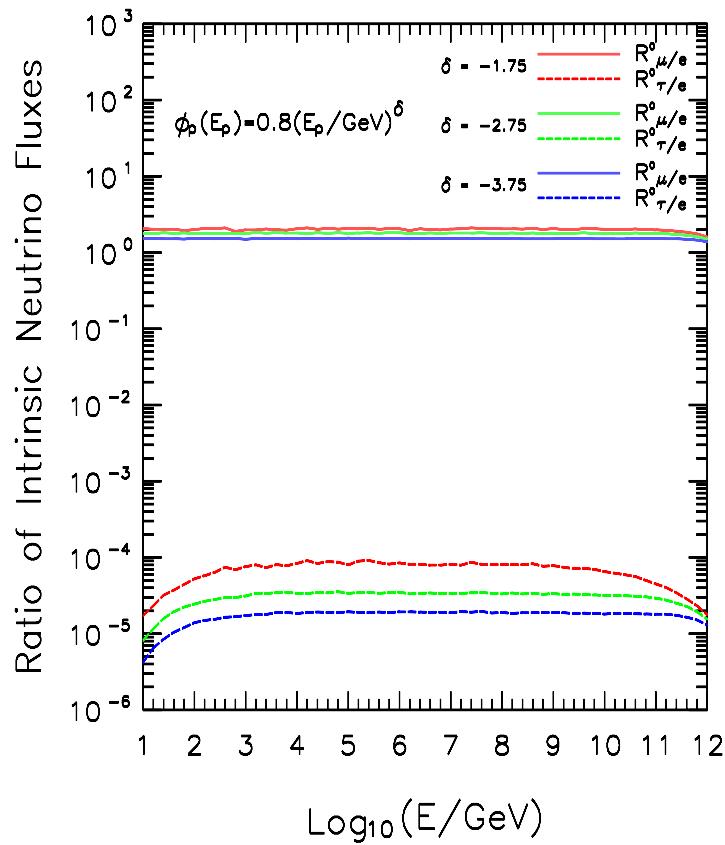


FIG. 3: The ratios of the intrinsic neutrino fluxes, the $R_{\mu/e}^0$ and the $R_{\tau/e}^0$, for three different cosmic-ray flux spectrum exponents as a function of the neutrino energy E . The two intrinsic astrophysical neutrino flavor ratios are essentially independent of the cosmic-ray flux spectrum exponent. The $\delta = -2.75$ case is also shown for comparison. More details are provided in the text.

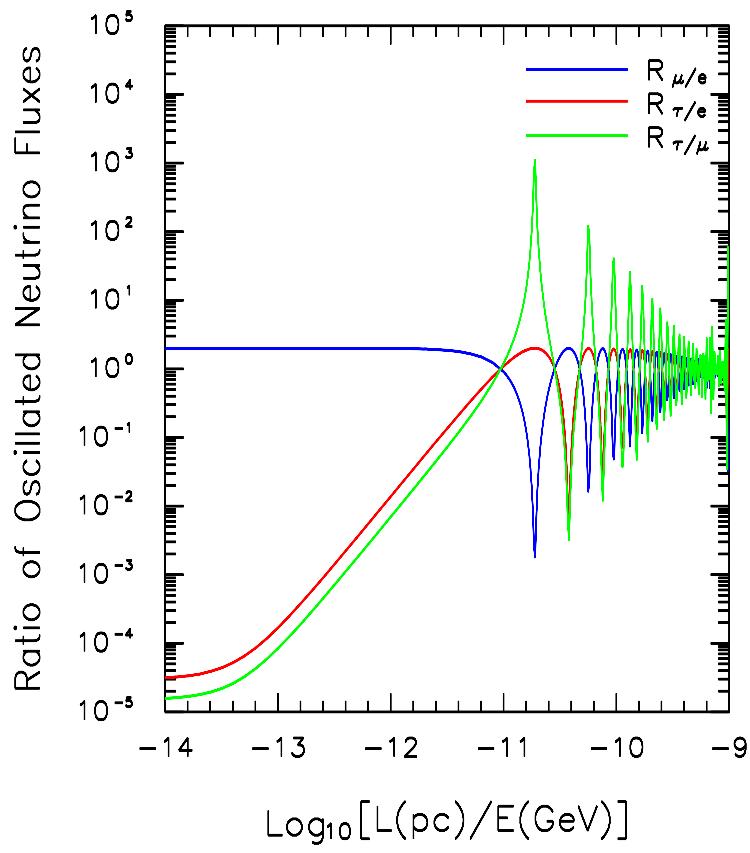


FIG. 4: The oscillated astrophysical neutrino flavor ratios as a function of the neutrino energy E . For $L(\text{pc})/E(\text{GeV}) \geq 10^{-10}$, the three ratios enter into a region of the relatively rapid and small amplitude oscillations centered at the abscissa axis value 1.

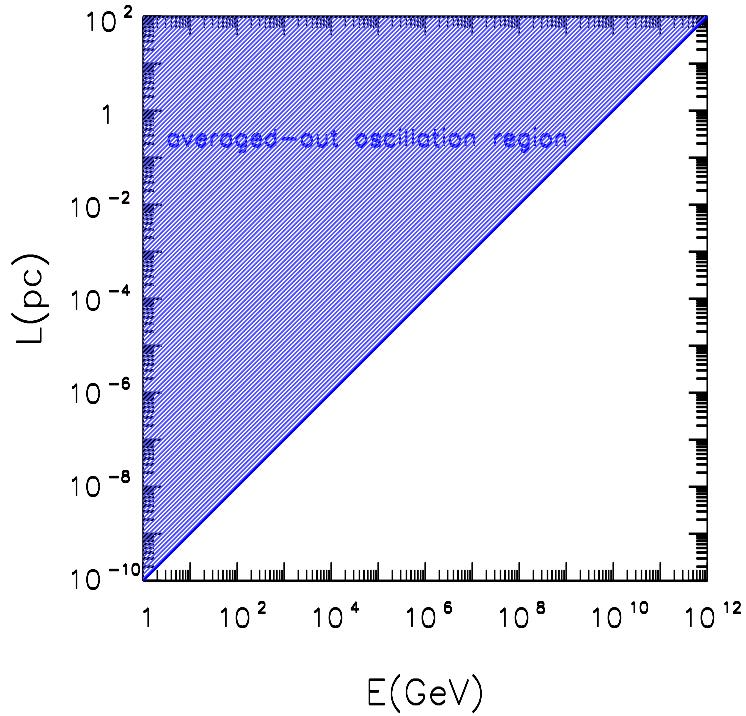


FIG. 5: The distance to the astrophysical neutrino source L in units of parsecs as a function of the neutrino energy E in GeV, using the criterion $L(\text{pc})/E(\text{GeV}) \geq 10^{-10}$. The shaded region indicates the L and the E value range for which the astrophysical neutrino flavor ratios may be 1:1:1 originating in the pp interactions.